Non-Abelian Fractional Quantum Hall States and Topological Quantum Computation

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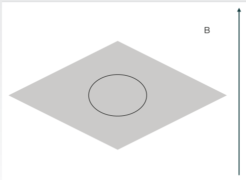
FRACTIONAL QUANTUM HALL

Fractional quantum Hall effect occurs in 2-dimensional electron gas exposed to perpendicular magnetic field. We perform exact diagonalization of finite-size systems to study the properties of the many-body state, incorporating the effect of finite width derived for specific samples and Landau level mixing corrections.

We consider the Hamiltonian of the form:

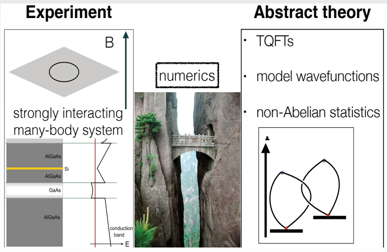
$$H = \sum_{L2} V_{L2} P_{L2} + \sum_{L3} V_{L3} P_{L3}$$



where P_{L2} (P_{L3}) is the operator projecting pairs (triples) of electrons to the state with the total angular momentum L2 (L3).

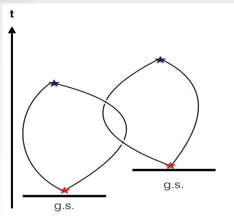
OUR MISSION

We perform realistic numerical simulations in order to understand if the desired and exotic non-Abelian physics of topological systems can actually be implemented in an accessible experiment.



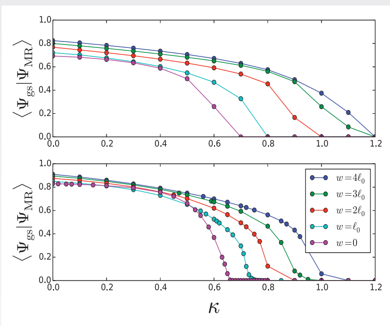
WHAT IS A TOPOLOGICAL SYSTEM?

- Described by a Topological Quantum Field Theory (TQFT) at low energy
- Correlators are invariant under manifold smooth transformations => local operators do nothing
- => not sensitive to noise
- quasiparticles fusion result (*) only depends on knots topology but not on the worldline details
- change knots topology = gate of information processing
- examlpes: fractional quantum Hall 5/2; 12/5 states



RESULTS SUMMARY

Finite thickness and LL mixing major effects



The major effects of Landau level mixing (kappa) and finite thickness (w) can be understood by looking at the overlaps presented here for the two setups (sphere and torus) and filling 5/2.

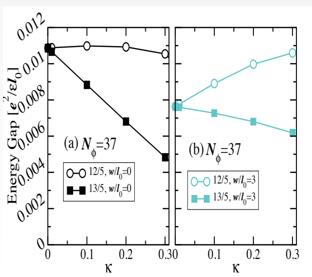
Increasing kappa slowly decreases the overlap and completely destroys the non-Abelian Pfaffian state after some critical value.

Finite thickness in general increases the overlap and increases the critical Landau level mixing strength kappa the MR state can survive.

Similar effect is observed for the other candidate anti-Pfaffian (not shown). One of the major results is the finding that Pfaffian is more robust to LL mixing.

LL mixing kills the 13/5 state

Energy gap for N=37 at 12/5 and 13/5 for w=0(a) and 3 (b). Similar results are obtained for smaller system sizes. LL mixing breaks particle-hole symmetry producing a larger energy gap for 12/5 compared to 13/5. The gap at w=3 for 12/5 increases with k while the 13/5 gap is suppressed. Hence, LL mixing strengthens the 12/5 FQHE for finite width while weakening 13/5.
This explains that unlike 12/5, 13/5 has never been observed in experiment.



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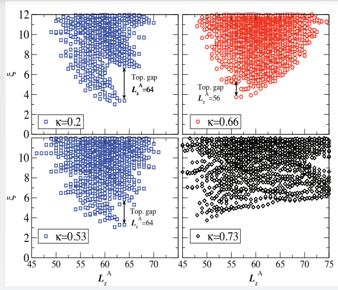
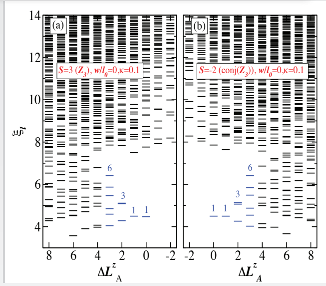
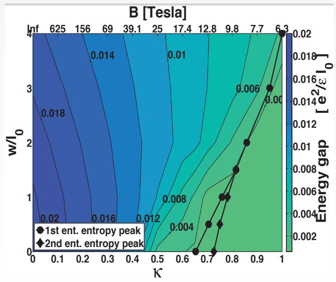
TOPOLOGICAL PROPERTIES AND PHASE DIAGRAM





Evolution of the state with increasing kappa can be well understood by looking at the entanglement spectrum above. At low kappa (0.2) we observe the signature of MR Pfaffian in the lower part of the spectrum separated by a sizable topological gap. With increasing kappa the gap shrinks and disappears around kappa=0.65. At kappa=0.66 a phase with opposite chirality sets in after a topological phase transition.

If the ground state is in the RR phase, the counting of the low-lying levels of the entanglement spectra will be related to the SU(2)3 TQFT describing the edge excitations. The counting of the low-lying levels for 13/5 and 12/5 for w=3 and k=0.1 matches the counting for Z3 and conj(Z3) non-Abelian wavefunctions, respectively. This means these FQHS are suitable for topological quantum computation under realistic conditions.



Phase diagram of the 5/2 FQHE is presented on the left. Color encodes the energy gap as a function of Landau level mixing and finite thickness.

A clear message to the experimental community seeking non-Abelian phase is to explore higher magnetic fields (low kappa) where the Pfaffian state is predicted to be more robust.

The area on the left from the black circles corresponds to the Pfaffian phase. The region between black circles and black diamonds is the phase with opposite chirality (possibly anti-Pfaffian phase).

ALGORITHM AND PARALLELISATION APPROACH

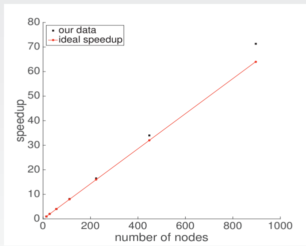
We exactly diagonalize the Hamiltonian and are interested in 1-2 lowest eigenvalues and eigenvectors. We use Lanzcos algorithm implementation from the ALPS project. We provide it with the matrix-vector multiplication function that is the most important and heavy part of the code.

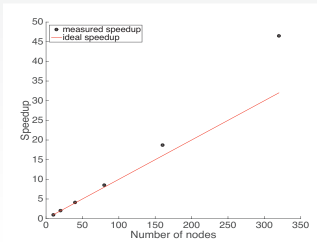
In our code we use all available memory to precompute part of or full matrix. Matrix is divided between nodes. Workload is parallelized using MPI+OMP. Increasing the number of nodes we can precompute larger fraction of matrix elements.

This explains the excellent scaling behaviour shown in the plots for a system with 115.195.490 basis states. Upper panel is on Piz Daint (32Gb per node). Lower panel is on Piz Dora (64Gb per node).

Larger memory per cpu allows us to save cpu time by running on smaller number of nodes.

We gratefully acknowledge the support from CSCS under projects s395 and s551 which made this study possible.





DISCUSSION AND OUTLOOK

This is the first realistic numerical study that includes both finite thickness and LL mixing effects.

We map out the first quantum phase diagram for the 5/2 fractional quantum Hall state and find a region at low Landau level mixing strength where the state is in the non-Abelian Pfaffian universality class.

This should guide the experiment in choosing the proper parameter regime.

Majorana fermions that live on the edge of the Pfaffian state could eventually be used as a building block for the error-free topological quantum computer.

For the 12/5 state we confirmed that it is in the non-Abelian Read-Rezayi state under realistic condition. This phase features Fibonacci anyons which unlike the Majorana fermions of 5/2 can be used to implement a *universal* topological quantum computer.

References:

[1] K. Pakrouski, M. R. Peterson, T. Jolicoeur, V. W. Scarola, C. Nayak, and M. Troyer, Phys. Rev. X 5, 021004 (2015).
[2] K. Pakrouski, M. Troyer, Y. Wu, S. Das Sarma and M. R. Peterson, PRB 94, 075108 (2016)